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Experimental Verification of Stability Region of Balancing a Single-wheel Robot: an Inverted Stick Model Approach

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***Abstract*—Gyroscopically manipulated lateral motion of a single-wheel based robot system has a parametric oscillation problem. In this paper, we analyze the stability regions for balancing the single-wheel robot system and then propose how to determine the offset range of its suppression controller. Since the roll motion is coupled with the pitch motion, the parametric oscillation problem should be analyzed. The dynamics model is parameterized as a Mathieu's equation from an inverted stick model. The stability region of a pitch oscillation parameter using the phase trajectory is analyzed from simulation. Simultaneously, its suppression control performance by determining the offset range is examined. Experimental studies are conducted to verify the analysis for the stable balancing control and to confirm the stability region based on the determined offset value.**

***Keywords*—stability region, single-wheel robot, parametric oscillation of an inverted stick**

I. INTRODUCTION

Stability of an autonomous mobile robot system is strongly dependent on the characteristics of a system structure such as the number and geometry of contact, center of gravity, static and dynamic stability, and inclination of terrain. In case of wheeled robotic systems, the minimum number of wheels required for the static stability is three. A two-wheel differential-drive system with a caster has a static stability all the time [1].

Two-wheel mobile robots have a difficulty of maintaining stable static posture. In case of a single-wheel robot system [2-6], the problem of the static stability is more difficult since it has a point contact property. Therefore, balancing control of two or one wheel mobile robots is quite challenging.

The successful balancing control performances of two-wheel mobile robots have been reported in the literature. A typical example is Segway [7]. To maintain balance of two-wheel mobile robots, control of the pitch angle in the longitudinal direction becomes the most important concern.

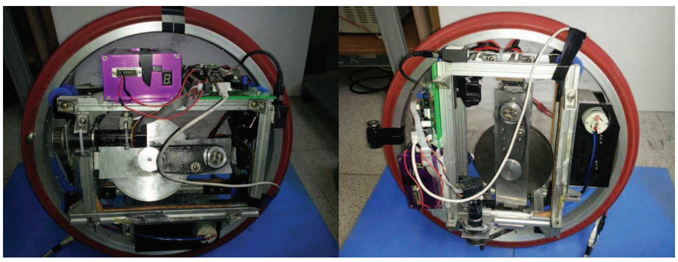
Since the robot has a single wheel, balancing control becomes more difficult. For the single-wheel robot, the roll angle control in the lateral direction becomes important and requires an extra actuation for the balancing performance.

Gyroscopically induced force has been used to maintain upright position of the single-wheel robots such as Gyrover and GYROBO [4,6]. Induced force by the combination of both movements of the roll and the pitch direction is used for the yaw direction. Therefore, pitch and roll motions are coupled and the coupled motion makes a robot system lose its stability gradually or rapidly during the balancing control.

GYROBO developed at Chungnam National University has two operational concepts as shown in Fig. 1. The horizontal mode (Fig. 1(a)) has a high-power low-instability characteristic and the vertical mode (Fig.1(b)) has a low-power high-instability characteristic. The horizontal mode has a yaw-pitch parametric oscillation characteristic but the vertical mode has a roll-pitch parametric oscillation characteristic. Research on the horizontal mode has been presented [6].

To achieve the lateral balancing control of the robot system, parametric properties of the system must be concerned. Especially when the vertical mode is used to save the power consumption, the parametric resonance becomes an important factor so that it must be taken into consideration due to the lack of the frictional absorption performance.

In this paper, the stability of the vertical mode of GYROBO is analyzed. Firstly, the dynamic equation of GYROBO is derived as the inverse stick model which has a vertically oscillation constraint [8,9]. The dynamic equation is parameterized as a Mathieu's equation to analyze the phase portraits. Secondly, the amplitude of the adequate offset value is determined by the numerical analysis. Thirdly, a vibration suppression control algorithm is proposed to suppress the parametric oscillation's enlargement. Finally, balancing control performance of GYROBO is tested empirically to verify the comparison between the calculated stability bandwidth and the determined offset range.



(a) Horizontal mode (b) Vertical mode

Fig. 1. Two balancing concepts of GYROBO.

II. SYSTEM CONFIGURATION

A. Modelling as an Inverse Stick

Two balancing modes of GYROBO are shown in Fig. 1. Fig 1 (a) has a yaw-pitch gyroscopic characteristic and Fig. 1 (b) has a roll-pitch gyroscopic characteristic. When the friction at the contact point is considered, Fig. 1 (b) is more sensitive in the pitch direction compared with Fig. 1 (a). However, the vertical mode has a power-efficiency advantage.

With the view of longitudinal direction, Fig. 1 (b) can be modelled as an inverted stick which has a vertical constraint as Fig. 2.

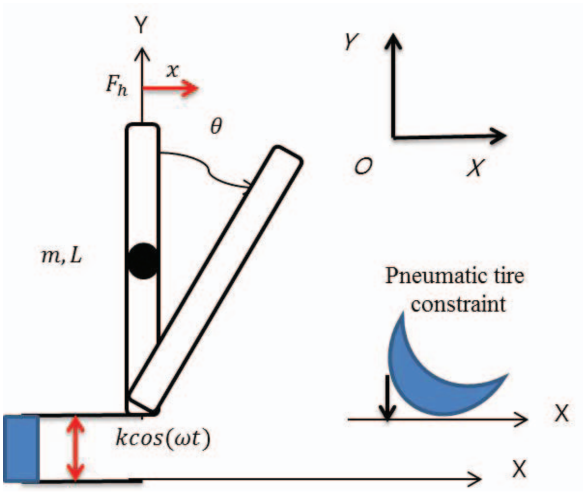


Fig. 2. Inverse stick model

The governing dynamic equation is given by

$$\frac{1}{6}mL\ddot{\theta}+(-mg+mk\omega^2\cos\omega t)\sin\theta=F_h$$
$$\ddot{\theta}+\left(-\frac{6g}{L}+\frac{6}{L}k\omega^2\cos\omega t\right)\sin\theta=0 \tag{1}$$

where θ is the balancing angle, ω is the angular velocity, m is the mass, L is the length, g is a gravity acceleration, F_h is an input force, and k is a constant.

Then, the equation can be parameterized as follows

$$\tau=\omega t,\omega_0{}^2=\frac{6g}{L} \tag{2}$$

where τ is the time constant and ω_0 is the natural frequency. Finally the governing equation can be modified as

$$\omega^2\frac{d^2\theta}{d\tau^2}+\left(-\omega_0{}^2+\frac{6}{L}k\omega^2\cos\tau\right)\sin\theta=0$$
$$\therefore\frac{d^2\theta}{d\tau^2}+\left(-\frac{\omega_0{}^2}{\omega^2}+\frac{6}{L}k\cos\tau\right)\theta=0 \tag{3}$$

The naturally parametric oscillation properties of the system are analyzed. Then, parameterized equations are given below which are known as a Mathieu type equation of parametric resonance [9,10].

$$\ddot{\theta}+(\alpha+\beta\cos\tau)\theta=0 \tag{4}$$

where, $\alpha=-\frac{\omega_0{}^2}{\omega^2},\beta=\frac{6k}{L}$.

B. Problem Statements

The stability problem of the parametric oscillation is given in terms of the parameter space (α,β) of equation (4). We assume that the parameter α has a value of '-1' in the paper [9]. The simplified description for the stability analysis can be shown as in Fig. 3. The maximum value of 'k' of the system is expected to be found in the following section.

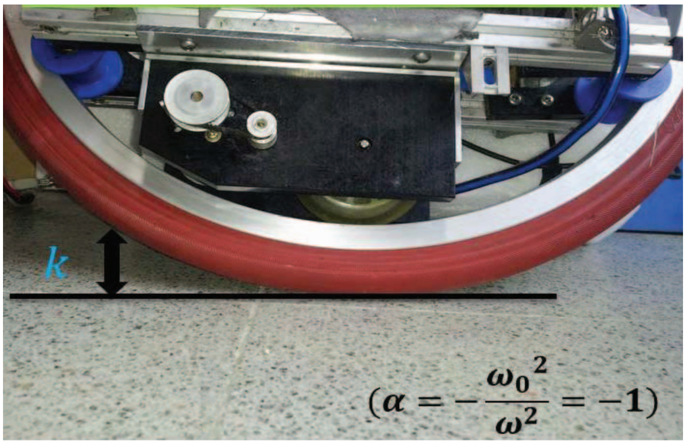


Fig. 3. Problem description

III. STABILITY ANALYSIS

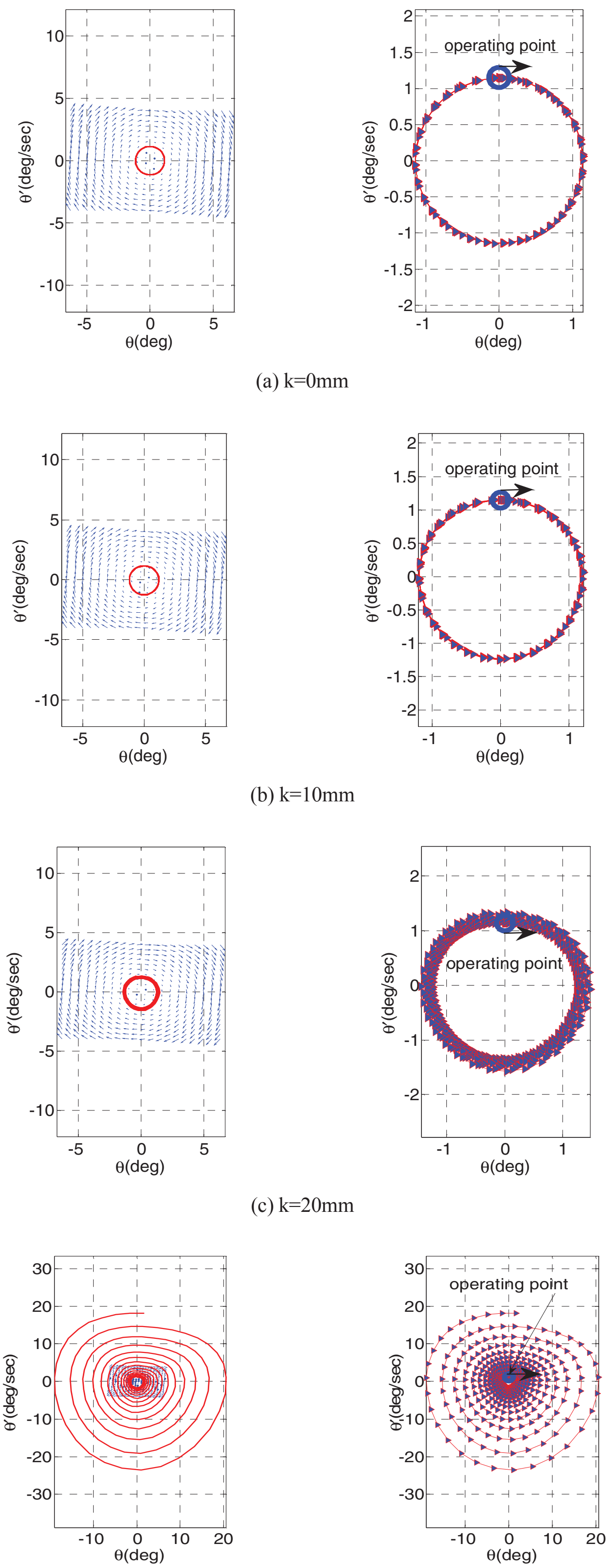
A. Phase and Time Trajectories

As a stability analysis method, we use a phase trajectory plot which shows all possible states of a given system. In this analysis, the system model has an originally oscillatory feature itself as shown in Fig. 4 (a). Fig. 4 (a) and (b) show a center node type stability characteristic which is stable. However, in case of Fig. 4 (c), although it looks like a center node, it has a gradually growing instability characteristic. In case of Fig. 4 (d), the system goes unstable exponentially.

Three properties of the stability analysis method can be definitely examined by inspecting the time plot about the 'theta' phase as shown in Fig. 5. We clearly see that the unstable behavior can be observed from Fig. 5 (d) when k = 50 mm.

Note that the parameter β is dependent on the value of 'L' and the value of 'k' from (4). When the value of the height of the robot system is known exactly, this property is not varying regardless of other parameters of dynamics.

The height 'L= 510(mm) is used in this simulation. We could determine the value of 'k' for the stability analysis using the simulation results as followings. The trajectory of phase using the phase portrait method is simulated and investigated as followings. The stability properties of the system are listed in Table 1.



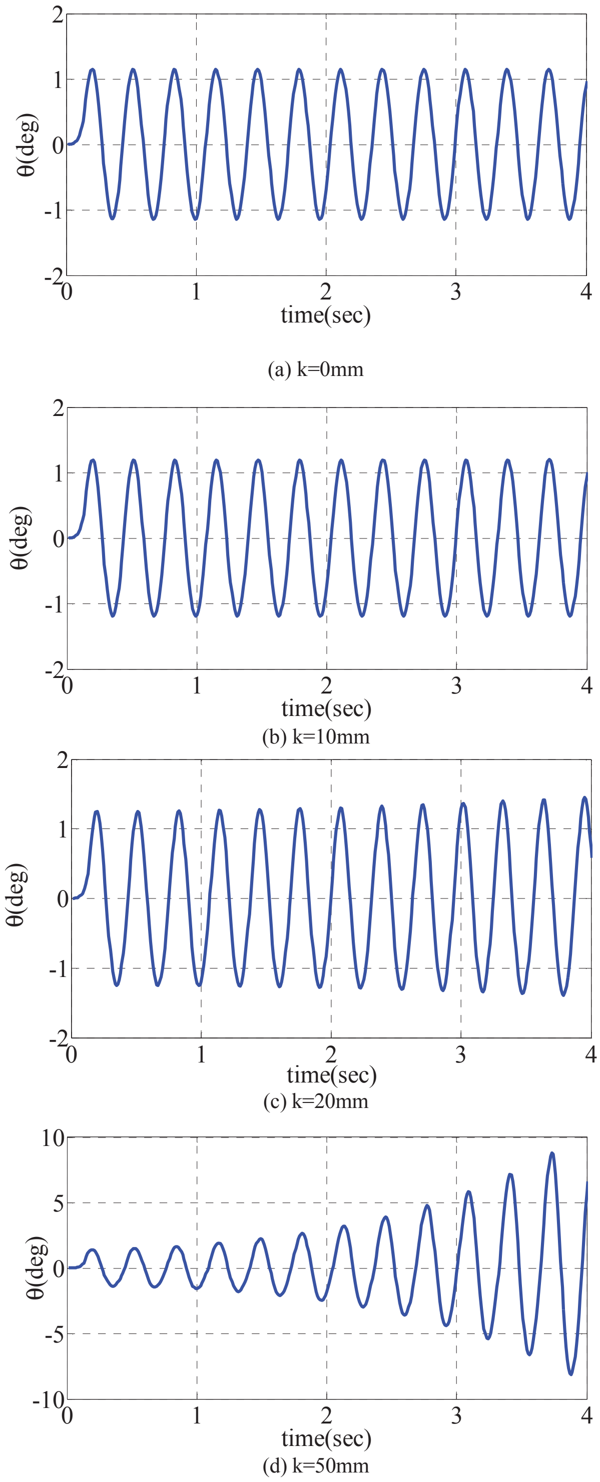
(a) k=0mm

(b) k=10mm

(c) k=20mm

(d) k=50mm

Fig. 4. Phase trajectories



(a) k=0mm

(b) k=10mm

(c) k=20mm

(d) k=50mm

Fig. 5. Time plots of θ

From the simulation of the stability analysis, the stability regions are found as listed in Table 1. We found the stable region when K < 10mm.

TABLE I. STABLE REGIONS OF K

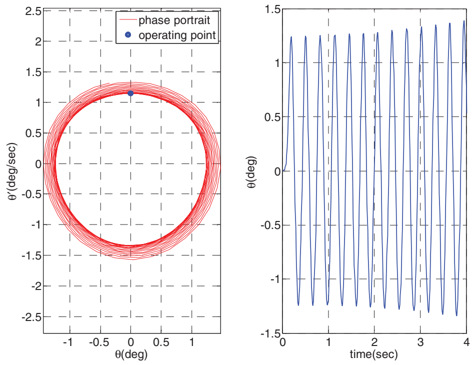
K(mm)	Stability Characteristics
k < 10	Stable
10 < k < 50	Gradually unstable but controllable
50< k	Rapidly unstable and uncontrollable

B. Offset Determination

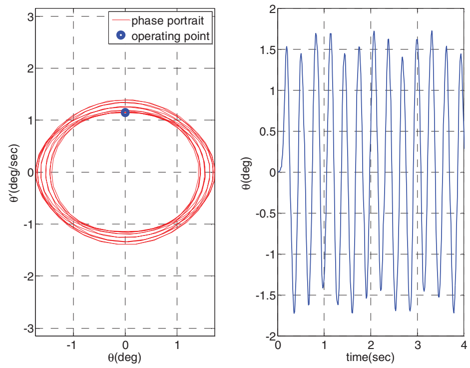
Next problem is how to choose the offset value A_{offset} in equation (5). The offset value is found by the numerical analysis of simulation. Two different values of k are simulated.

$$\ddot{\theta} + (-1 + 12k\cos\tau)(\theta - A_{offset}\sin\theta) = 0 \qquad (5)$$

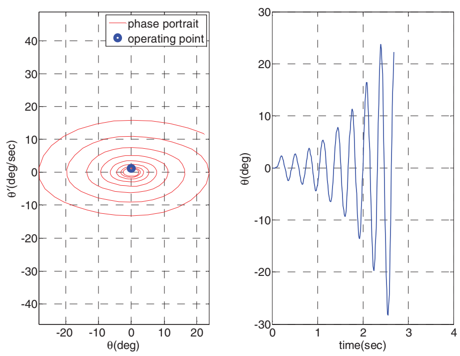
(1) Case 1: k=20



(a) A_{offset}=0



(b) A_{offset}=0.375

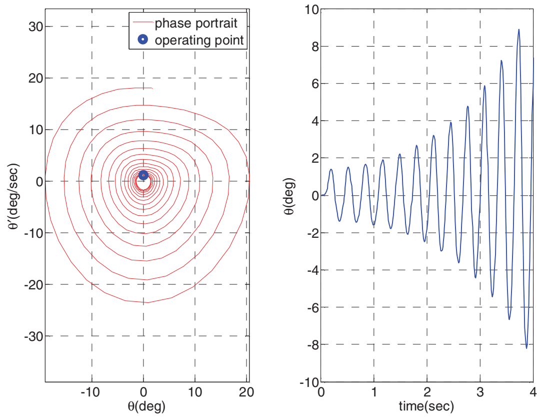


(c) A_{offset}=0.75

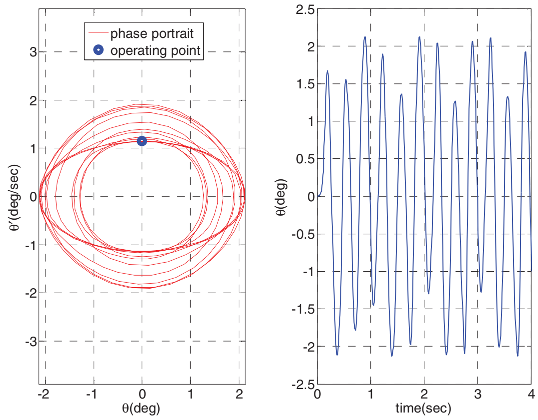
Fig. 6. Phase plots with A_{offset}.

(2) Case 1: k=50

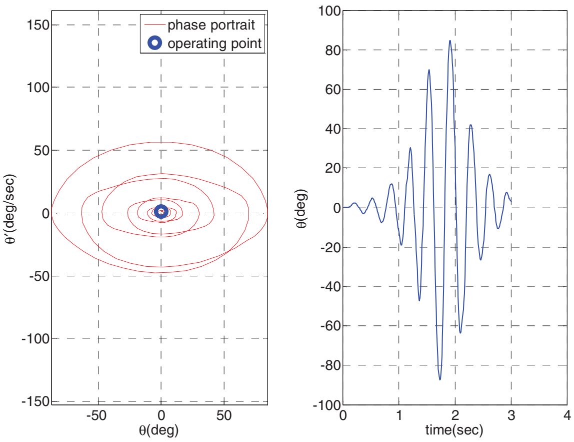
And then, we set the parameter 'k' as 50 and examine the phase plot characteristics.



(a) A_{offset}=0



(b) A_{offset}=0.375



(c) A_{offset}=0.75

Fig. 7. Phase plots with A_{offset}.

From the numerical analysis results, the offset value nearby 0.5 degrees is chosen.

IV. Experimental Verification

A. Controller Deign

The control law for the controllable parameter ‘k’ is designed. A linear PD-controller with a nonlinear offset compensation for the balance control is used. The offset angle value is added to the reference trajectory so that it can minimize the vibration.

$$u = K_p\theta_e + K_d\dot{\theta}_e \tag{6}$$

where K_p, K_d are controller gains.

Fig. 8 shows the balancing control block diagram.

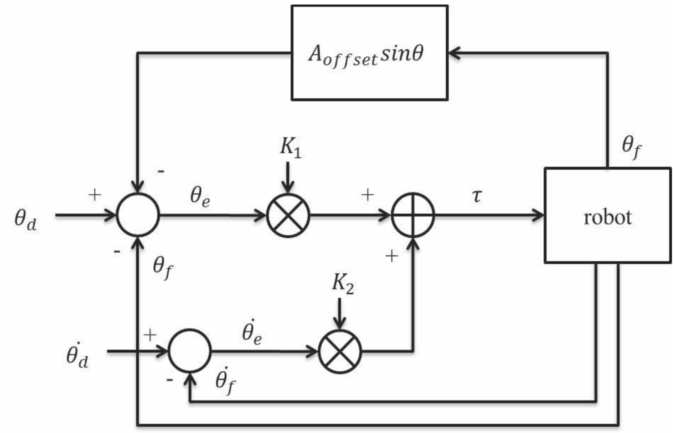


Fig. 8. Control block diagram

A_{offset} =0.5 is chosen as an offset value which is found by the numerical analysis. However, this control logic is not enough because the gimbal position is not considered. To enhance the control logic, the gimbal position and the gyroscopic torque are analyzed as shown in Fig. 9.

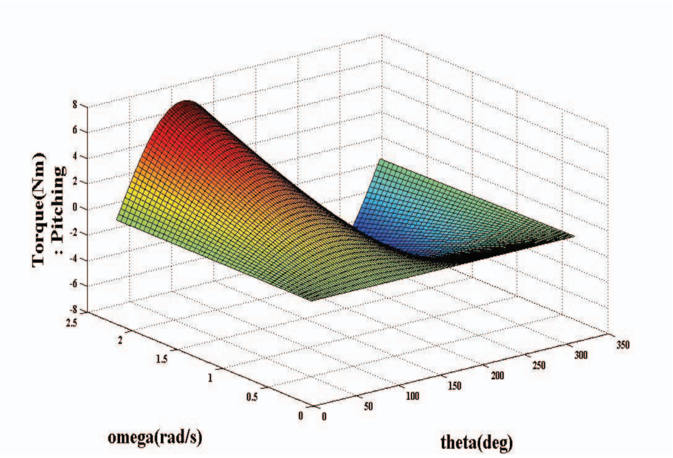


Fig. 9. Gimbal position – gyroscopic torque relation

The offset value can be calculated by considering the gimbal’s position as below

$$A_{offset} = 0.5 \times \frac{gimbal\_angle}{2\pi} \times roll\_bandwidth \tag{7}$$

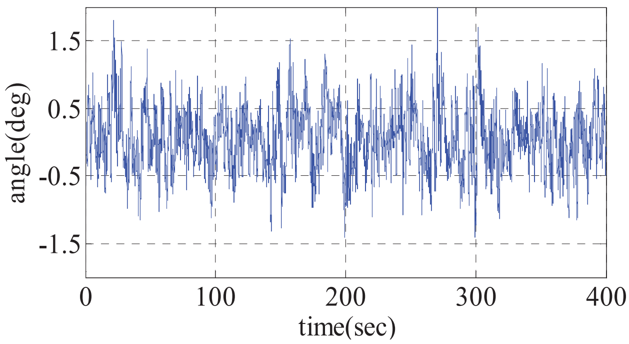
Since the roll bandwidth is found to be 4, equation (7) becomes

$$A_{offset} = 0.5 \times \frac{gimbal\_angle}{2\pi} \times 4 \tag{8}$$

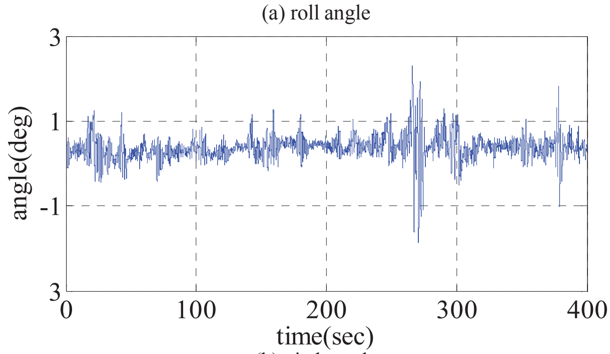
In (8), when the position of the gimbal is 90 degrees, the whole gyroscopic torque is going toward the pitch direction as shown in Fig. 9. This state means that the magnitude of the parametric oscillation hits the ceiling. Therefore, the compensation value ‘k’ is selected as the largest value of 0.5. However, when the gimbal position stays nearby the center, there is a very weak oscillation of parameter in the system. So, in this case, the oscillation controller is not required.

D. Experimental Verification

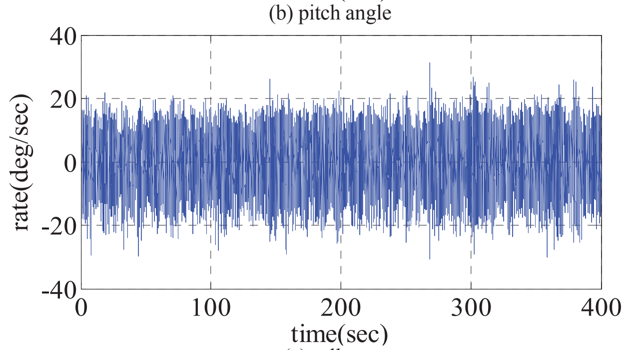
The balancing control of the vertical mode of GYROBO has been tested to verify the stability region found from the analysis in the previous sections. Fig. 10 shows the balancing performance results. The roll and pitch angles are well maintained at balance. The balancing angle error is within ±1.5 degrees.



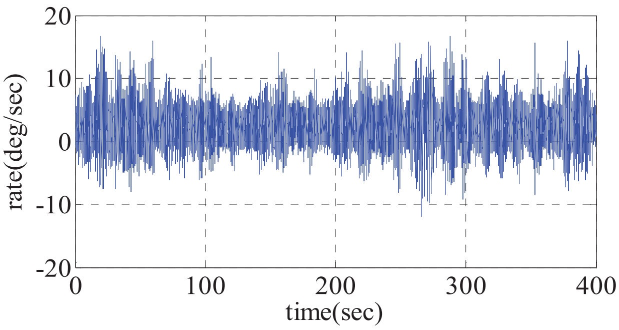
(a) roll angle



(b) pitch angle



(c) roll rate



(d) pitch rate
Fig. 10 Balancing performance

V. Conclusions

The parametric oscillation problem of the vertical mode of the single-wheel robot system for balancing control was investigated by the inverse stick model. Mathieu's equation having parametric characteristic was simulated and the controllable regions were determined. Based on the physical and systematical analysis of the hardware structure, the anti-oscillating control method of determining offset values was proposed. The gimbal's position during the control was considered as well. Experimental studies confirmed the feasibility of applying the proposed method for stable balancing in the vertical mode. As a practical way of controlling a point contact single-wheel system, the proposed method can be one of the solutions.

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References

[1] Siegwart, Roland, Illah Reza Nourbakhsh, and Davide Scaramuzza. *Introduction to autonomous mobile robots*. MIT press, 2011.

[2] Y. S. Xu, Kwok Wai Au and H. B. Brown, "Dynamic Mobility with Single-Wheel Configuration", *The international Journal of Robotics Research*, pp. 728-738, 1999

[3] Y. Ou and Y. Xu, "Stabilization and Line Tracking of the Gyroscopically stablized Robot", *IEEE International Conference on Robotics & Automation*, May 2002.

[4] Y. Xu and K. W. Au, "Stabilization and Path Following of a Single Wheel Robot", *IEEE/ASME Transactions on Mechatronics*, vol. 9, No. 2, pp. 407-419, June 2004.

[5] P. K. Kim, J. H. Park, and S. Jung. "Experimental studies of balancing control for a disc-typed mobile robot using a neural controller: GYROBO." *Intelligent Control (ISIC), 2010 IEEE International Symposium on*. IEEE, 2010.

[6] J. H. Park, and S. Jung. "Development and control of a single-wheel robot: Practical Mechatronics approach." *Mechatronics* vol. 23, no. 6, pp. 594-606, 2013.

[7] Segway, http:// segway.com

[8] Insperger, Tamas. "Stick balancing with reflex delay in case of parametric forcing." *Communications in Nonlinear Science and Numerical Simulation*, vol.16, no.14, pp. 2160-2168, 2011.

[9] Moreno-Ahedo, Luis, Joaquin Collado, and Carlos Vazquez. "Parametric Resonance Cancellation Via Reshaping Stability Regions: Numerical and Experimental Results." *IEEE Transactions on Control Systems Technology,* vol. 22, no.2, pp. 753-760, 2014.

[10] Broer, H. W., et al. "The inverted pendulum: a singularity theory approach." *Journal of differential equations,* vol.157, no. 1, pp. 120-149 1999.